Experimental Analysis of Low Earth Orbit Satellites due to Atmospheric Perturbations

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***Abstract*—** A satellite is expected to move in the orbit until its life is over. This would have been true if the earth was a true sphere and gravity was the only force acting on the satellite. However, a satellite is deviated from its normal path due to several forces. This deviation is termed as orbital perturbation. The perturbation can be generated due to many known and unknown sources such as Sun and Moon, Solar pressure, etc. This paper discusses the study of perturbation of a Low Earth satellite orbit due to the presence of aerodynamic drag. Numerical method (Runge - Kutta fourth order) is used to solve the Cowell's equation of perturbation, which consists of ordinary differential equation.

***Keywords*— Low Earth Orbit (LEO), Cowell's method, atmospheric perturbation, orbital elements, numerical method (RK4)**

I. INTRODUCTION

Satellite is a space vehicle which is used for different purposes like communication, weather forecasting, surveillance, etc. so, for each purpose the design, working and the altitude the satellite to be placed, is different for different types of satellites which depends on the purpose of the satellite. A perturbation is actually a deviation from real or expected motion. It can be small or large depending upon the type of perturbation and the type of orbit the satellite is in. The satellites when launched, must account for the possible perturbations that can be corrected otherwise in long run the small perturbations will add up and can cause a complete failure in the satellite's expected mission.

A. Kepler's Laws

There are three basic Kepler's laws which are the fundamental laws followed by any satellite [6] are given below:-

1) A satellite follows an elliptical path to its center of attraction.
2) In equal times, the areas swept by the radius vector of a satellite are the same
3) The periods of any two satellites around the same planet are related to their semi major axis.

II. TYPES OF ORBITS

Majorly, there are two major divisions in orbit types; viz. circular orbits and elliptical orbits

A. **Circular orbit** is an orbit which has eccentricity equal to zero. It is not perfectly circular, the general name for any orbit that is not highly elliptical is circular and it comprises of geostationary, polar, sun-synchronous, and equatorial orbits.

B. **Equatorial Orbit** is an orbit which moves along the line of the Earth's equator.

C. **Geosynchronous orbit (GSO)** is nearly at an altitude of 36000 Km from earth surface. The period of revolution of a satellite in this orbit is equal to period of rotation of earth, but its orbit is not equatorial with an orbital period of one sidereal day, matching the Earth's sidereal rotation period.

D. **Geostationary orbit / Geosynchronous equatorial orbit (GEO)** is a circular orbit above the Earth's equator, following the direction of the Earth's rotation; has an orbital period equal to the Earth's rotational period (one sidereal day), and thus appears motionless, at a fixed position in the sky, to ground observers. Nearly at an altitude of 36000 Km from earth surface.

E. A **Polar orbit** has an inclination of approximately 90^0 degrees to the equator.

F. A **Sun-Synchronous orbit (SSO)** changes slowly in time with the planet moving around the Sun, and in time with the planet's rotation so that the spacecraft is always at the same angle to the Sun.

G. The geocentric orbit with altitudes from 100 to 1,500 km is L**ow Earth Orbit (LEO)**.

H. The geocentric orbit ranging in altitude from 5,000 km to just below geosynchronous orbit at 35,786 km is **Medium Earth Orbit (MEO)** or intermediate circular orbit.

I. The geocentric orbit above the altitude of geosynchronous orbit 35,786 km is known as **High Earth Orbit.**

J. A **Highly Elliptical Orbit (HEO)**, also called an eccentric orbit, is in the shape of an ellipse. Elliptical orbits are varying in speed & have eccentricity between zero and one.

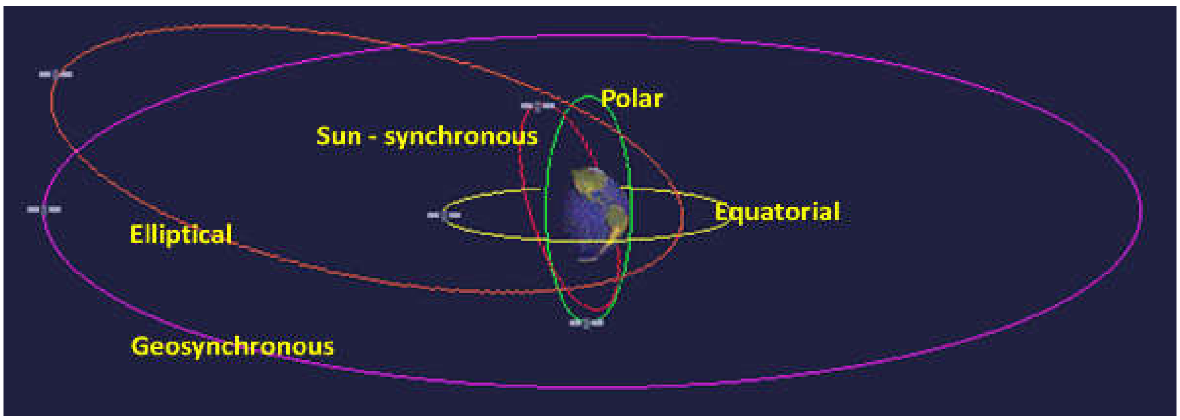

Fig. 1 Types of Orbits

III. KEPLERIAN OR ORBITAL ELEMENTS

A. ***Semi-Major Axis***: It is a geometrical parameter of an elliptical orbit. It can be computed as

$$A_{(Semi\ major\ Axis)} = (A_{(Apogee)} + P_{(Perigee)}) / 2 \quad (1.1)$$

B. ***Eccentricity***: The orbit eccentricity *e* is the ratio of the distance between the center of the ellipse and the center of the Earth to the semi-major axis of the ellipse.

$$E_{(eccentricity)} = (A_{(Apogee)} - P_{(Perigee)}) \text{ by } (A_{(Apogee)} + P_{(Perigee)}) \quad (1.2)$$

$e_{(eccentricity)} = 0$ *; for circle*

$e_{(eccentricity)} < 1$ *; for ellipse*

$e_{(eccentricity)} = 1$ *; for parabola*

$e_{(eccentricity)} > 1$ *; for hyperbola*

C. ***Apogee***: Apogee is the point on the satellite orbit that is at the farthest distance from the center of the Earth. The Apogee distance *A* can be computed as

$$A_{(Apogee)} = a\,(1+e) \quad (1.3)$$

D. ***Perigee***: Perigee is the point on the satellite orbit that is nearest to the center of the Earth. The Perigee distance *P* can be computed as

$$P_{(Perigee)} = a\,(1-e) \quad (1.4)$$

E. ***Inclination***: The inclination *i,* is the angular distance of the orbital plane (in fact, the normal of the orbital plane) w.r.t a reference plane (for example, if the orbit is around the Earth, the reference plane is the equatorial or ecliptic plane). Normally stated in degrees. $0° < i < 180°$.

F. ***Ascending and Descending Nodes***: The satellite orbit cuts the equatorial plane at two points: the first, called the descending node (N1), where the satellite passes from the northern hemisphere to the southern hemisphere, and the second, called the ascending node (N2), where the satellite passes from the southern hemisphere to the northern hemisphere.

G. ***Vernal Equinox*: ** The line of intersection of the Earth's equatorial plane and Earth's orbital plane that passes through the center of the Earth is known as the line of equinoxes. The direction of this line with respect to the direction of the Sun on 21 March determines a point at infinity called the Vernal Equinox (Y).

H. ***Right Ascension of Ascending Node* (*RAAN*):** The right ascension of the ascending node tells about the orientation of the line of nodes, which is the angle made by the line joining the ascending and descending nodes, with respect to the direction of the vernal equinox. It is also known as the longitude of the ascending node Ω.

I. ***Argument of Perigee*: ** The argument of perigee ω, is the angle measured from the ascending node (or line of nodes) to the perigee in the counter clockwise sense. $0^o < \omega < 360^o$. It determines the orientation of the orbit inside its plane.

J. ***True Anomaly*: ** The true anomaly υ, is the angular distance, measured in the orbital plane, from the occupied focus (i.e., Earth's center) from the perigee to the current location of the satellite (orbital body). Countered in the direction of movement of the satellite. In degrees. $0^o < \upsilon < 360^o$.

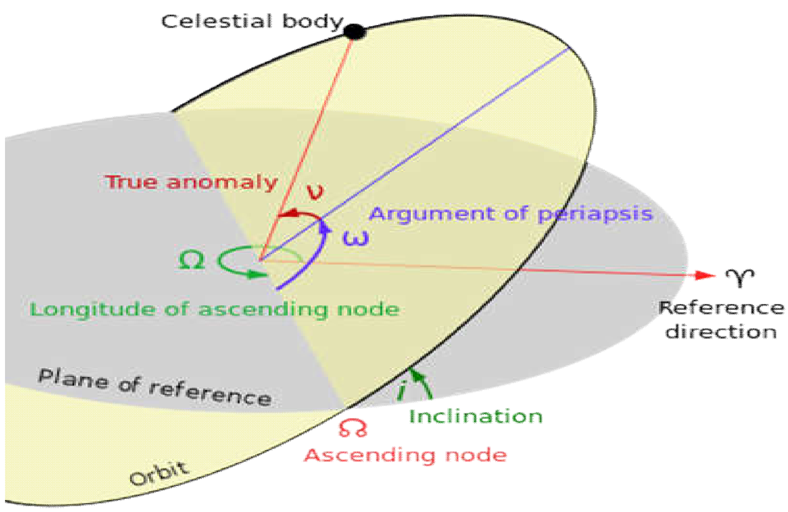

Fig. 2 Orbital Elements

IV. PERTURBING TORQUES

As the satellite is placed in its nominal orbit, it experiences disturbances which are caused due to different forces [2], [7] which are discussed below:-

A. J2 gravity model or due to non-sphericity of Earth
B. Atmospheric Drag
C. Solar Radiation Pressure
D. Magnetic Disturbance
E. Third Body effect
F. Ocean Tide

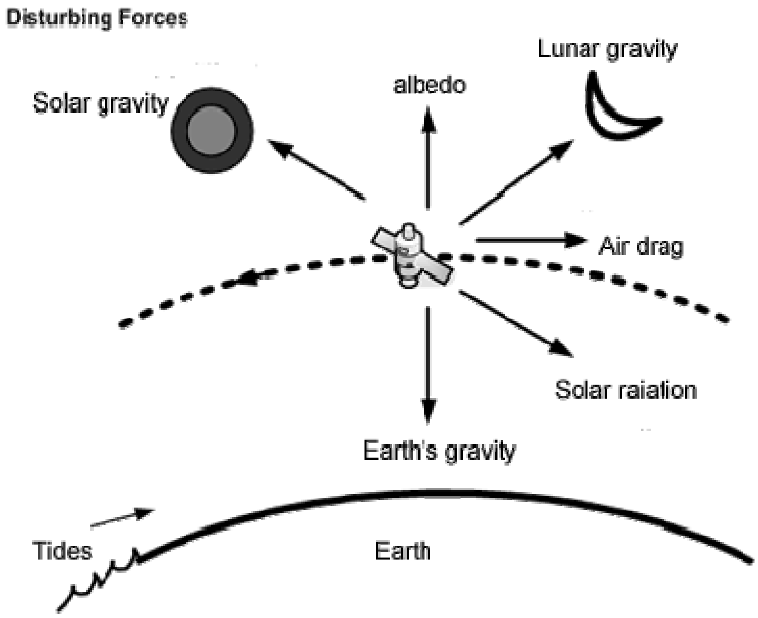

Fig. 3 Disturbing Forces [8]

The main effective disturbance torques for satellites are discussed below:-

A. Solar Radiation

Solar radiation pressure produces a force on the satellite related to its distance to the sun and its effect is more at high altitudes. While determining the resulting acceleration caused by solar radiation the surface area of the satellite should faces the sun which is an important condition.

The force of solar pressure can be given as:

$$p_{(solar\ radiation)} = SRC / C = \frac{1353}{3\times 10^8}\frac{W/m^2}{m/s} = 4.51 \times 10^{-6}\ N/m^2 \quad (1.5)$$

Where, SRC: is the solar radiation constant = SF 1353 W/m^2

C: is the speed of the light c = 3×10^8 m/s

The torque due to the solar radiation is given by:

$$\tau_{(Solar\text{-}radiation)} = -\,p_{(solar\ radiation)} \times C_{(r)} \times A_{\odot} \times (C_{(psr)} - C_{(g)}) \quad (1.6)$$

Where A_{Θ} is the exposed area to the Sun, c_r is the reflectivity, $c_{(psr)}$ is the estimated centre of pressure and c_g is the centre of gravity.

B. Aerodynamic Drag

At low altitude, satellites will be influenced by the air density. The effect is dependent on the area and shape of the surface. This effect may reduce the velocity of the satellite. The aerodynamic torque is given as:

$$\tau_{aero} = F_{(aero)}(u_v \times (C_{(ps)} - C_{(g)})) \tag{1.7}$$

$$F_{aero} = \frac{1}{2}\rho v^2 C_{(drag)} A_{(inc)} \tag{1.8}$$

Where,
ρ - Atmospheric density (kg /m^3)
$A_{(inc)}$ - Area perpendicular to u_v (m^2)
u_v - Unit vector in velocity direction
$C_{(drag)}$ - Drag coefficient
V - Velocity of satellite (m/s)
$C_{(ps)}$ - Centre of pressure
$C_{(g)}$ - Centre of gravity

C. Gravity Gradient Torque

Any non-symmetrical object in the orbit is affected by a gravitational torque because of the variation in the Earth's gravitational force over the object. There are many mathematical models for gravity gradient torque. The most common one is derived by assuming homogeneous mass distribution of the Earth.

$$\tau_{(gravity)} = (3\mu/R_0^3)u_e \times (Iu_e) \tag{1.9}$$

where $\mu = 3.986 \times 10^{14}$ m^3/s^2 is the Earth's gravitational coefficient, R_0 is the distance from Earth's centre (m), I is the inertia matrix and finally, u_e is the unit vector towards nadir.

D. Magnetic Disturbance Torque

This torque is resulted from the interaction of the geomagnetic field and spacecraft's residual magnetic field. If M is the sum of all magnetic moments in the satellite, the torque acting on the satellite=

$$T^{(m)} = M_{(magnetic\ field)} \times B_{(geomagnetic\ field\ vector)} \tag{1.10}$$

Where, B is the geomagnetic field vector. M is caused by satellite generated current loops, permanent magnets or induced magnets.

V. COWELL'S METHOD

This method was discovered by P. H. Cowell in the early 20th century. Cowell's method is the simplest and most direct method to calculate perturbations of all other methods [1]. The application of Cowell's method is simply to write down the two body equation of motion, including all possible perturbations and then to integrate them step-by-step numerically.

For the two body problem with perturbations, the equation would be [4]

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = \overline{a_p} \tag{1.11}$$

The above equation is a second order differential equation. Numerically, we can also write above equation into first order differential equation as below

$$\dot{\bar{r}} = \bar{v} \qquad \dot{\bar{v}} = \overline{a_p} - \frac{\mu}{r^3}\bar{r} \tag{1.12}$$

where r and v are the radius and velocity vectors of a satellite with respect to the larger central body. Equation (1.12) can be broken down into vector components

$$\dot{x} = v_x \qquad \dot{v}_x = a_{px} - \frac{\mu}{r^3}x$$

$$\dot{y} = v_y \qquad \dot{v}_y = a_{py} - \frac{\mu}{r^3}y \tag{1.13}$$

$$\dot{z} = v_z \qquad \dot{v}_z = a_{pz} - \frac{\mu}{r^3}z$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

The perturbing acceleration is the summation of all the perturbing forces mentioned earlier, but our main consideration for a low earth orbit satellite is atmospheric drag because the satellite is very close to earth's atmosphere i.e., at an altitude of about 300 to 400 km from earth surface. Therefore we can write

$$\overline{a_p} = \overline{a_{ad}} \tag{1.14}$$

Where a_{ad} = acceleration due to atmospheric drag

Therefore, by substituting equation (1.14) into equation (1.11), we get

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = \overline{a_{ad}} \tag{1.15}$$

Using Cowell's method, equation (1.15) can be written as

$$\dot{\bar{r}} = \bar{v} \qquad \dot{v} = a_{ad} - \frac{\mu}{r^3}r \tag{1.16}$$

The analytical formulation of the perturbation (r and v) can be found by applying numerical integration methods to above equation i.e., Runge – Kutta method. The Runge – Kutta method is designed for greater accuracy and they provide the advantage of requiring only the function values at some selected point in the subinterval, while there are other methods as well such as Euler's method, which less efficient in practical problems since it requires h i.e., step size or interval to be small for obtaining reasonable accuracy. Therefore, the sensible choice is Runge – Kutta method which is more accurate.

VI. RESULTS

Runge - Kutta method is used to solve the Cowell's equation of perturbation, which consists of ordinary differential equation (ODE). To solve the ODE, the initial conditions i.e., initial position and velocity of the satellite is required.

The satellite used as a reference is ISS (International Space Station) with [8]

Initial position:	X = -3754.66380	
	Y = -5060.64117	kilometre
	Z = -2517.73324	
And Initial velocity:	V_x = 5.356846188	
	V_y = -1.332207337	kilometre / sec
	V_z = -5.317508141	

And with a Vector Time (GMT) = 2018 / 092 / 13:19:31.516

The variation of position with respect to time where position is mentioned in kilometre (km) and time is considered in days, is shown below in Fig. 4. The position of the satellite is computed for a time period of one month i.e., 30 days with an appropriate interval.

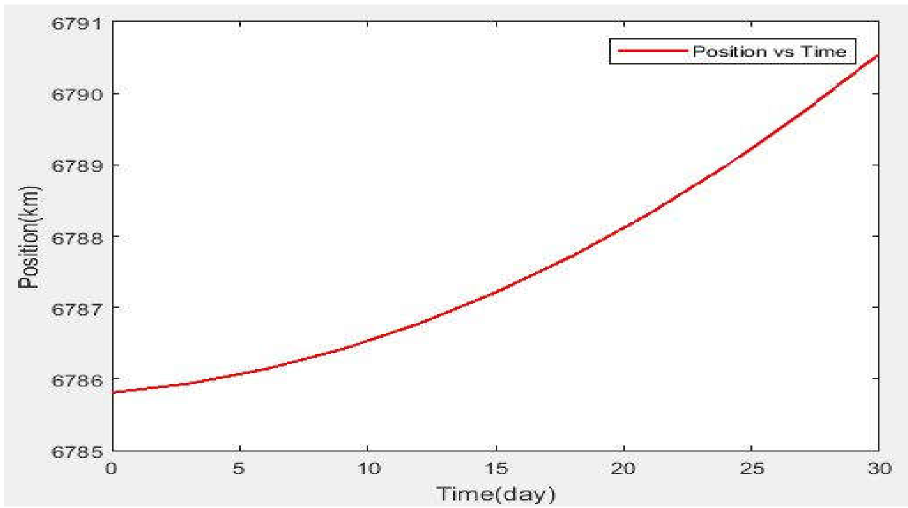

Fig. 4 Change in Position with respect to Time

Similarly, the variation of Velocity (in km/s) and Semi-major axis (in km) with respect to Time (in days) is mentioned below in Fig. 5 and Fig. 6 respectively. The computed velocity and semi-major axis is for a time period of 30 days. These values are computed by the help of numerical method (Runge – Kutta fourth order method).

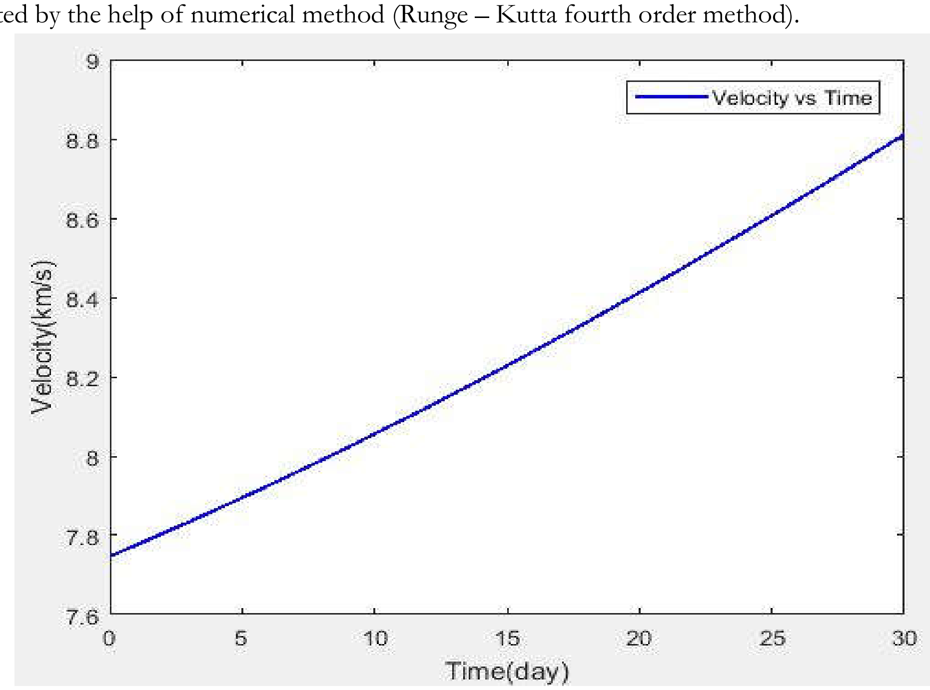

Fig. 5 Change in Velocity with respect to Time

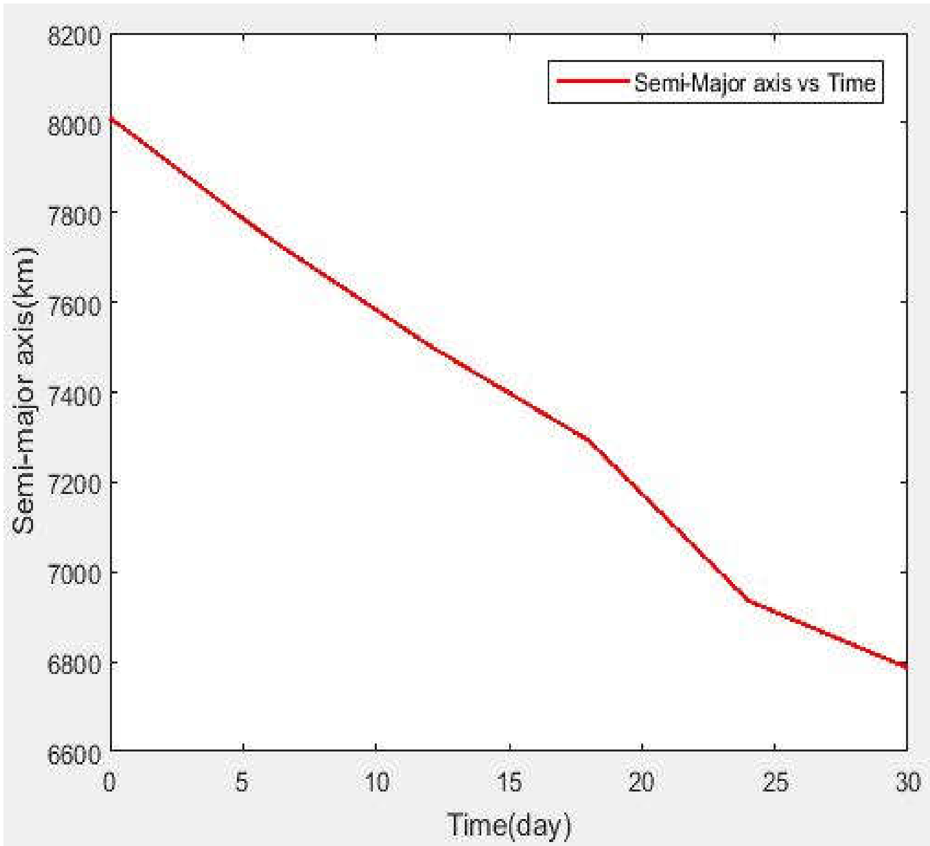

Fig. 6 Change in Semi-major axis with respect to Time

The path travelled by satellite can be visualised by the help of GMAT (General Mission Analysis Tool) software in which the Cartesian state vectors i.e., position and velocity or keplerian elements are used as the input at a particular point of time, known as Epoch and the path of the satellite over the Earth surface is the output. The Fig. 7 shows the ground track plot and Fig. 8 shows the orbit view of the satellite corresponding to the initial position and velocity.

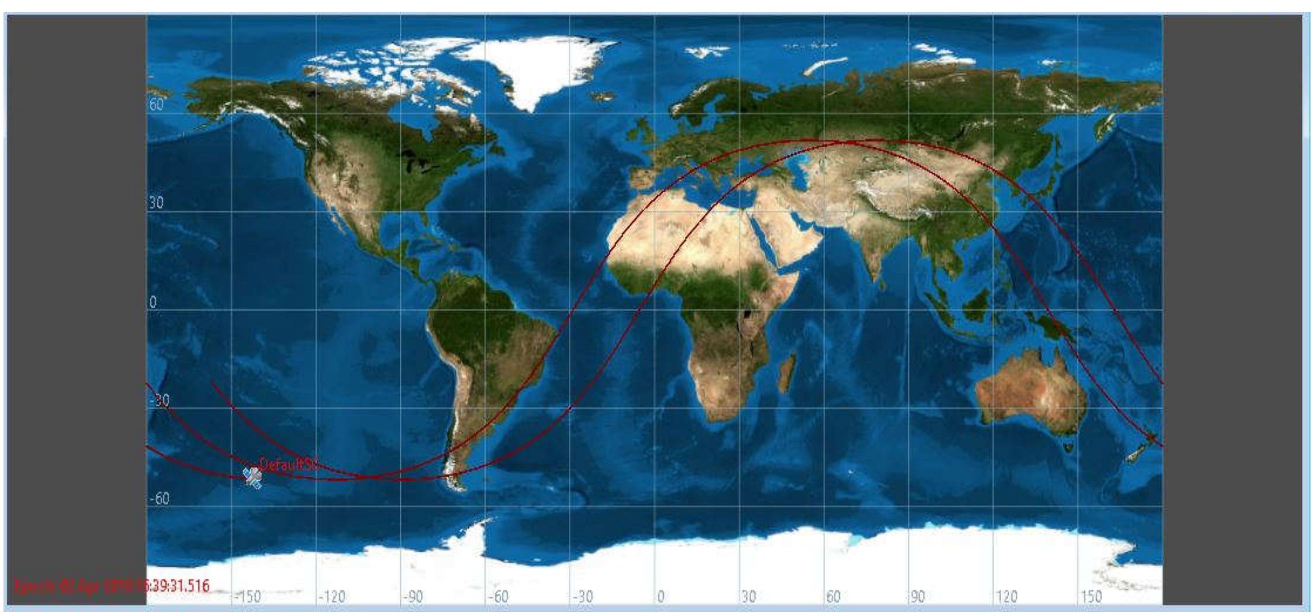

Fig. 7 Ground Track Plot at Epoch - 02 Apr 2018 16:39:31.516

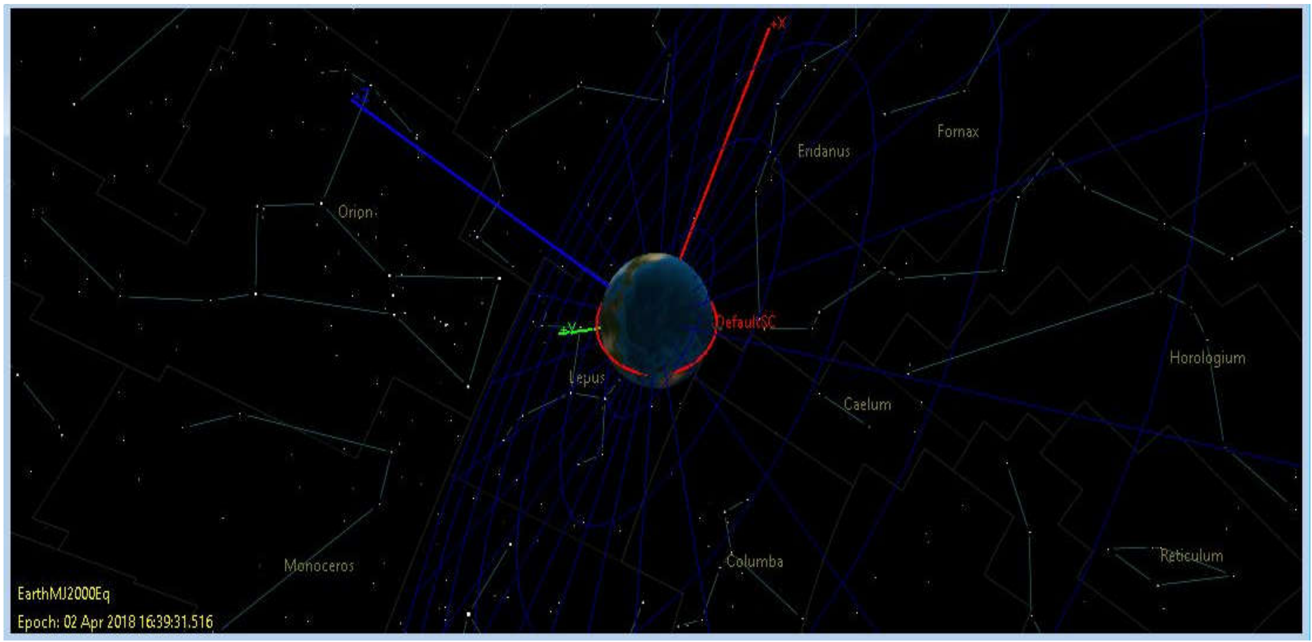

Fig. 8 Orbit View at Epoch - 02 Apr 2018 16:39:31.516

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Fig. **9** shows the ground track plot and Fig. 10 shoes the orbit view corresponding to final values of position and velocity which is calculated by the help of Runge – Kutta method.

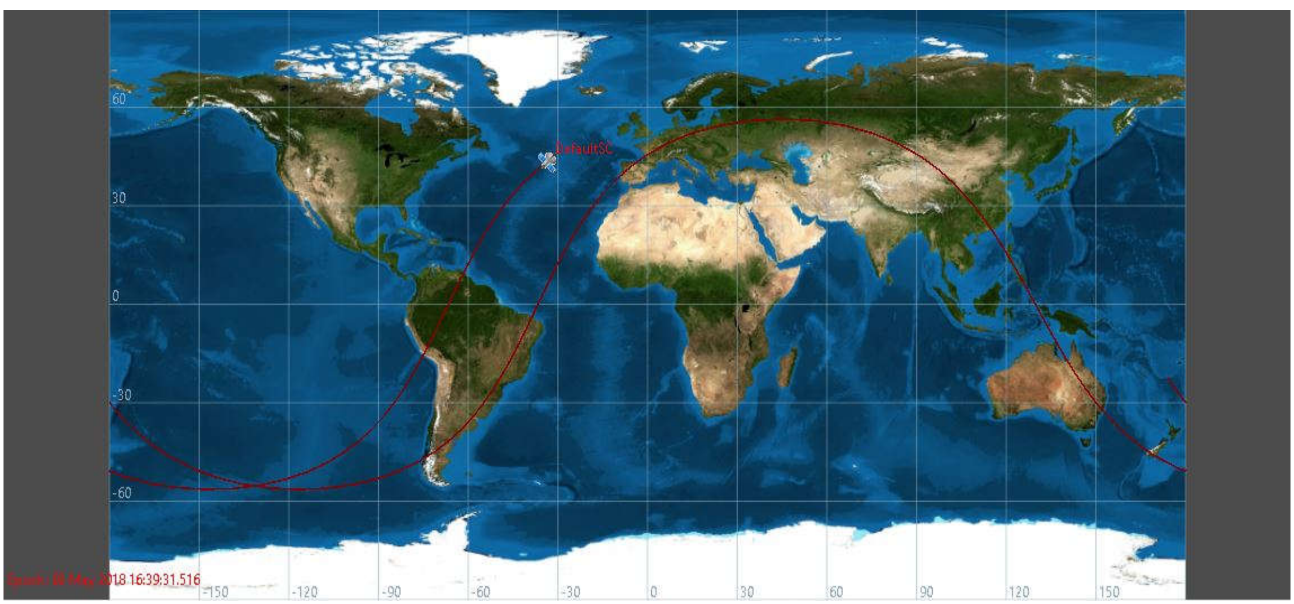

Fig. 9 Ground Track Plot at Epoch - 02 May 2018 16:39:31.516

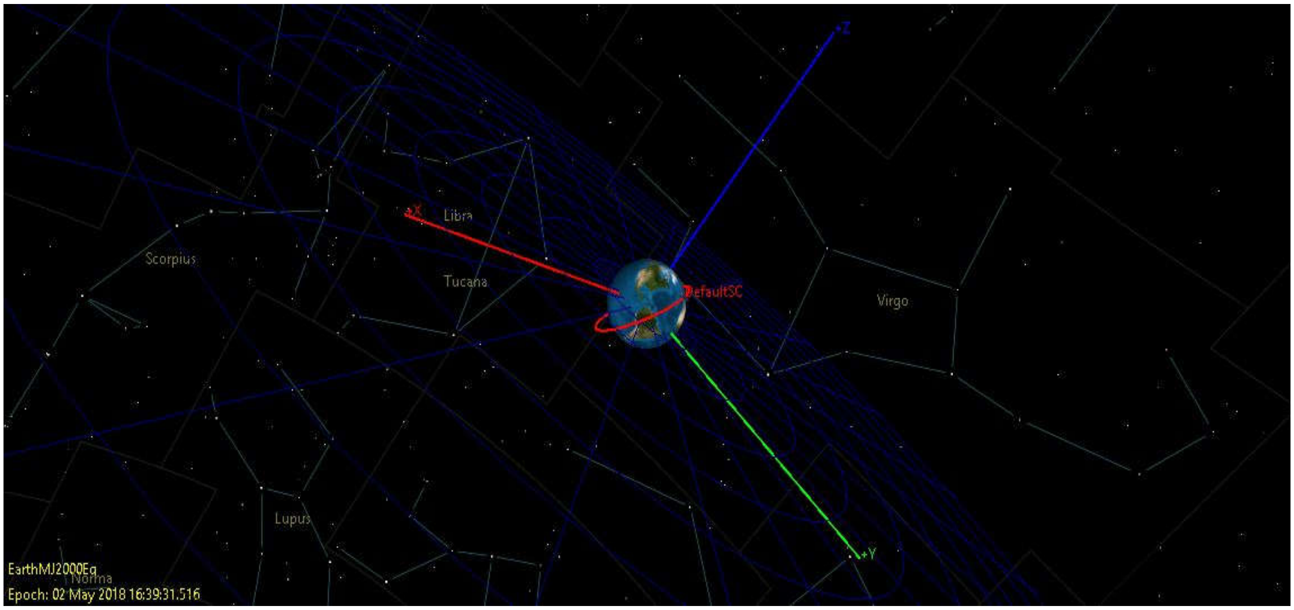

Fig. 10 Orbit View at Epoch - 02 May 2018 16:39:31.516

VII.CONCLUSION

The perturbed magnitudes obtained in this paper are quite large for a time period of 30 days. Therefore, in order to have a precise calculation of satellite position and velocity at any particular time, the perturbation accelerations cannot be neglected. Some acceleration such as solar radiation pressure, Sun and Moon gravity, etc. are neglected because of their negligible effect on the Low Earth orbit satellites while they are the major source of perturbation in Geosynchronous or Geostationary orbit satellites. Cowell's method is used to calculate the perturbations because it is the most straight forward method and it is quite accurate compared to Encke's method. The Semi-major axis reduces with time, which concludes that the satellite's orbit is reducing i.e., Orbital Decay is present which will reduce the lifetime of the satellite and the satellite will finally fall back to the Earth's surface.

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REFERENCES

[1] *S. P. Shuster, "A Survey and Performance Analysis of Orbit Propagators for LEO, GEO, and Highly Elliptical Orbits," All Graduate Theses and Dissertations, Utah, 2017.*

[2] *K. C, "LEO SATELLITES: ATTITUDE DETERMINATION AND CONTROL COMPONENTS ; SOME LINEAR ATTITUDE CONTROL TECHNIQUES," Ankara, 2006.*

[3] *M. Eshagh and M. Najafi Alamdari, "Perturbations in orbital elements of a low earth orbiting satellite," Journal of the Earth & Space Physics, vol. 33, no. 1, pp. 1-12, 2007.*

[4] *K. A. Muhammad Shamsul and R. Radzuan, "A STUDY OF PERTURBATION EFFECT ON SATELLITE ORBIT USING COWELL'S METHOD," Penang.*

[5] *A. Tewari, Atmospheric and Space Flight Dynamics, New York: Birkhauser Boston, 2007.*

[6] *A. E. Roy, Orbital Motion, IOP Publishing Ltd, 2005.*

[7] S. Alam and M. Yasir, "Satellite Attitude and Orbital Dynamics Simulator," Journal of Space Technology, vol. 1, no. 1, pp. 40-44, 2011.

[8] nptel, [Online]. Available: http://nptel.ac.in/courses/105104100/lectureB_9/B_9_5forces.htm.

[9] "ISS Trajectory Data," NATIONAL AERONATICS AND SPACE ADMINISTRATION, [Online]. Available: https://spaceflight.nasa.gov/realdata/sightings/SSapplications/Post/JavaSSOP/orbit/ISS/SVPOST.html.