A note on generalized fuzzy ideals of seminearring modules

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Abstract: In this paper, we consider the algebraic system seminearring which is a generalization of both a semiring and a nearring. A seminearring S is an algebraic system with two binary operations: usual addition and usual multiplication such that S forms a semigroup with respect to both the operations, and satisfies the right distributive law. The set of natural numbers with the usual operations of addition and multiplication is a semiring. Every ring is a nearring. In particular, the set of real numbers, the set of complex numbers and the set of integers are rings as well as near rings with respect to the usual addition and multiplication. A natural example of a seminearring is obtained by considering the operations usual addition and composition of mappings on a set of all mappings of an additive semigroup S into itself. In this paper we define fuzzy ideal of a seminearring module and a preliminary result related to this notion is proved.

Key words: smearing, nearing, seminearring, s-ideal, seminearring module.

1.1 INTRODUCTION

The notion of a fuzzy subset was first introduced by Zadeh [15]. It is a method for representing uncertainty. The term Fuzzy means "imprecise", "unclear", "indistinct". Fuzzy set is a generalization of the notion classical set. To distinguish between Fuzzy sets and classical sets, we refer to the latter as 'crisp' sets. Zadeh has defined a fuzzy set as a generalization of a characteristic function, wherein the degree of membership of an element is more general than merely yes or no denoted by 1 and 0 respectively. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. Rosenfeld introduced the notion of fuzzy sets in the realm of Group theory.

1.2 Definition: A non-empty fuzzy subset μ (that is, $\mu(x) \neq 0$ for some $x \in S$) of a seminearring S is called a fuzzy s-ideal With thresholds $\alpha, \beta \in [0, 1]$, $\alpha < \beta$ if it satisfies

(i) $\alpha \vee \mu(x + y) \geq \beta \wedge (\mu(x) \wedge \mu(y))$

(ii) $\alpha \vee \mu(xy) \geq \beta \wedge (\mu(x) \vee \mu(y))$

This is also called generalized fuzzy s-ideal or (α, β) fuzzy s-ideal.
In case if $\alpha = 0$ and $\beta = 1$,
Then this definition coincides with the usual fuzzy s-ideal.
In this note we consider the notion
(α, β) fuzzy s-ideal where $\alpha, \beta \in [0, 1]$, $\alpha < \beta$.

1.3 Definition: A fuzzy s-ideal μ of S is called a fuzzy s-k- ideal of S
if for all $x, y, z \in S$,

$$x + y = z \text{ implies that}$$
$$\alpha \vee \mu(x) \geq \beta \wedge \{\mu(y) \wedge \mu(z)\}$$

1.4 Definition: Let μ be a fuzzy subset of a seminearring S. Then the set defined by

$$\mu_t = \{x \in S \mid \alpha \vee \mu(x) \geq \beta \wedge t,\ t \in [0,1]\},$$

$$\beta \geq t.$$

is called the level subset of S with respect to μ.

1.5 Definition: Let S be a seminearring with multiplicative identity 1. An additive semigroup (M, +) with neutral element zero is called a left S-seminearmodule

if there exist a function

$S \times M \rightarrow M$ such that

(s, m) is denoted by sm, then

(i) (r + s)m = rm + sm

(ii) (rs)m = r(sm)

(iii) 1.m = m

(iv) r0 = 0m = 0 for all r, s $\in$ S

and m $\in$M.

Similarly, if the elements of S act on right, then we refer it as right S-seminearmodule.

In general, we denote M for a seminearring module.

1.6 Definition: A fuzzy set μ of a seminearring module M is called a fuzzy S-sub-seminearmodule of M if it satisfies the following conditions

for $\alpha < \beta$ and $\alpha, \beta \in [0, 1]$

(i)$\alpha \vee \mu(x+y) \geq \beta \wedge \{\mu(x) \vee \mu(y)\};$

and

(ii) $\alpha \vee \mu(ay) \geq \beta \wedge \mu(y)$ for all

$$x, y \in M, a \in S.$$

2.1 Theorem: Let M be S-seminearmodule and $\mu : M \rightarrow [0, 1]$ a fuzzy subset of M.

Then the following conditions are equivalent:

a) μ is a fuzzy S-sub-seminearmodule of M;

and

b) μ_t is S-sub-seminearmodule of M for all $t \in [0, \mu(0)]$.

Proof: First we will prove that

$(a) \Rightarrow (b)$:

Let t with $0 \leq t \leq \mu(0)$.

Since $\mu(0) \geq t$,

we have that $0 \in \mu_t = \{x \mid \mu(x) \geq t\}$.

So μ_t is a non-empty subset of M.

Let $x, y \in \mu_t$. Then $\mu(x) \geq t$

and

$$\mu(y) \geq t \Rightarrow \alpha \vee \mu(x+y) \geq$$

$$\beta \wedge \{\mu(x) \wedge \mu(y)\}$$

$$= \min\{t, t\} = t \Rightarrow x + y \in \mu_t.$$

So $(\mu_t, +)$ is a subsemigroup of (M, +).

Let g $\in$ M.

Further, for any $y \in M, a \in S,$

and

$t \in [0, 1]$

We have $\alpha \vee \mu(ay) \geq \beta \wedge \mu(y)$ for

all $x, y \in M, a \in S.$

This means ay $\in \mu_t$.

μ_t is S-sub-seminearmodule of M

for all $t \in [0, \mu(0)]$.

Now we will prove that

$(b) \Rightarrow (a)$:

Let $x, y \in S$

and

write $t = \{\mu(x) \wedge \mu(y)\}$.

Now

$\alpha \vee \mu(x) \geq t, \alpha \vee \mu(y) \geq t$

$\Rightarrow x,y \in \mu_t \Rightarrow x + y \in \mu_t$

Since μ_t is an ideal,

$\Rightarrow \mu(x + y) \geq t$

(by the definition of μ_t).

Therefore $\alpha \vee \mu(x + y) \geq \beta \wedge \{\mu(x) \wedge \mu(y)\}$.

Therefore μ is a fuzzy

S-sub-seminearmodule of M.

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