MHD FREE CONVECTION FLOW THROUGH A POROUS MEDIUM BOUNDED BY A VERTICAL SURFACE WITH HEAT AND MASS TRANSFER EFFECTS

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ABSTRACT

BACKGROUND

The study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. The growing need for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction.

OBJECTIVE

In the present paper, an analysis is carried out to study the effects of radiation on a free convection flow bounded by a vertical surface embedded in porous medium with constant suction under the influence of uniform magnetic field in the presence of a homogenous chemical reaction and viscous dissipation.

RESULT

The non-dimensional governing equations are solved analytically and the expressions for velocity, temperature, concentration fields are obtained. Also the expressions for skin friction, rate of heat and mass transfer are derived. The effect of important physical parameters on the velocity, temperature and concentration are shown graphically and also discussed the skin-friction coefficient, Nusselt number and Sherwood number are shown in tables.

CONCLUSION

Velocity increases as Grashof number or Modified Grashof number or Eckert number or porosity parameter increases. Temperature increases as the permeability parameter increases while it decreases as radiation parameter increases. Concentration decreases with increasing chemical reaction parameter or Schmidt number in both Cases of the study.

KEY WORDS

MHD, Free convection, Vertical surface, viscous dissipation, Porous medium, Heat and mass transfer.

1. INTRODUCTION

The growing need for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction. The presence of a foreign mass in water or air causes some kind of chemical reaction. This may be present either by itself or as mixtures with air or water. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, for example, polymer production, manufacturing of ceramics or glassware and food processing. A chemical reaction can be codified as either a homogenous or heterogeneous process. This depends on whether it occurs on an interface or a single phase volume reaction. A reaction is said to be of first order if its rate is directly proportional to the concentration itself [Cussler, 1988]. The effect of chemical reaction on heat and mass transfer in a laminar boundary layer flow has been studied under different conditions by several authors like Gbadeyan et al. (2011) Chamkhaet al, (2014), Kumar et al. (2013), Raju et al. (2014), Rao et al. (2013, 2015), Reddy et al. (2012, 2015, 2016, 2017). Radiation effect is another phenomenon which cannot be ignored. In view of the importance of radiation several authors studied this effect. For example Mamatha et al. (2015), Nageswaramma et al. (2015), Raju et al. (2014), Rao et al. (2013), Reddy et al. (2016), Seshaiah et al. (2013), Umamaheswar et al. (2013) and Vidyasagar et al. (2015) considered the presence of radiation in their studies.

The objective of the present paper is to analyze the radiation effects on MHD free convection flow through porous medium bounded by a vertical surface in presence of homogeneous chemical reaction. The dimensional less equations of continuity, linear momentum, energy and diffusion, which govern the flow field are solved by using a perturbation technique. The behavior of the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

2. MATHEMATICAL ANALYSIS

We consider an electrically conducting, radiating, viscous incompressible fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface discussed in two cases viz. Case (I):uniform temperature and concentration. Case (II):Constant heat and mass flux. In both cases, the x^* - axis is taken along the plate in vertical upward direction and the y^* - axis normal to it. A uniform magnetic field of strength B_0 is assumed to be applied in a direction perpendicular to the surface against to the gravitational field. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall Effect are negligible. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence the Soret and Dufour are negligible. Now, under the usual Boussinesq's approximation, the governing boundary layer equations of the problem are:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$v^*\frac{\partial u^*}{\partial y^*} = \vartheta\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T(T^* - T^*_\infty) + g\beta_C(C^* - C^*_\infty) - \frac{\sigma B_0^2}{\rho}u^* - \frac{\vartheta}{k^*}u^* \tag{2}$$

$$v^*\frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_P}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\vartheta}{C_P}\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y^*} \tag{3}$$

$$v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - k_c(C^* - C^*_\infty) \tag{4}$$

It is assumed that the level of species concentration is very low, hence the heat generated due to chemical reaction is neglected. The relevant boundary conditions are given as follows.

Where u^*, v^* are the velocity components in x^*, y^* directions respectively. ρ -the fluid density, ν - the kinematic viscosity, c_p - the specific heat at constant pressure, g -the acceleration due to gravity, β and β^* - the thermal and concentration expansion coefficient respectively, B_0 - the magnetic induction , α - the fluid thermal diffusivity, - the permeability of the porous medium, T - the dimensional temperature, C - the dimensional concentration, k -the thermal conductivity, μ - coefficient of viscosity, D - the mass diffusivity, K_c - the chemical reaction parameter and F is the radiation parameter

The boundary conditions for the velocity, temperature and concentration fields are:

$$u^* = 0,\ \frac{\partial T^*}{\partial y^*} = -\frac{q}{k},\ \frac{\partial C^*}{\partial y^*} = -\frac{q_w}{D} \text{ at } y = 0 \tag{5}$$

$$u^* \to 0, T^* \to T_\infty, C^* \to C_\infty \text{ as } y \to \infty \tag{6}$$

From the continuity equation (1), it is clear that suction velocity normal to the plate is either a constant or function of time. Hence, it is assumed in the form

$$v^* = constant = -v_0 \qquad (7)$$

Where u^* is the plate velocity, T_w and C_w are the wall dimensional temperature and concentration respectively, T_∞ and C_∞ are the free stream dimensional temperature and concentration respectively, v_0 - the constant. In the optically thick limit, the fluid does not absorb its own emitted radiation that is there is no self absorption, but it does absorb radiation emitted by the bounadries. Cookey et al. [26] showed that in the optically thick limit for a non gray gas near equilibrium as given below.

$$\frac{\partial q_r}{\partial y^*} = 4(T^* - T_w^*)\int_0^\infty K\lambda_w w(\frac{de_{b\lambda}}{dT^*}) / d\lambda = 4I_1(T^* - T_w^*) \qquad (8)$$

In order to write the governing equations and the boundary condition in dimension less form, the following non-dimensional quantities are introduced.

$$y = \frac{v_0 y^*}{v},\ u = \frac{u^*}{v_0},\ \theta = \frac{T^* - T_\infty^*}{\left(\frac{\vartheta q}{v_0 k}\right)},\ C = \frac{C^* - C_\infty^*}{\left(\frac{\vartheta q_w^*}{v_0 D}\right)},\ \mathrm{P_r} = \frac{\mu c_p}{k},$$

$$G_r = \frac{\vartheta^2 g\beta_T q}{k v_0^4},\ G_m = \frac{\vartheta^2 g\beta_C q_w^*}{k v_0^4}, S_c = \frac{\vartheta}{D},\ F = \frac{4I_1\vartheta}{k v_0^2}\ K_C = \frac{\vartheta K_C^*}{v_0^2},\ E = \frac{v_0^3}{C_P\left(\frac{q}{k}\right)\vartheta},\ M = \frac{\sigma B_0^2\vartheta}{\rho v_0^2},\ k = \frac{v_0^2 k^*}{\vartheta^2}$$

(9)

In view of Equations (6) to (8), Equations (2) to (4) reduced to the following dimensionless form.

$$u'' + u' = -G_r\theta - G_m C + M_1 u \qquad (10)$$

Where $M_1 = M + \frac{1}{k}$ $\theta'' + P_r\theta' = -P_r E u'^2 + F\theta$

(11)

$$C'' + S_c C' = K_c S_c C \qquad (12)$$

Where G_r, G_m, M, K, Pr, F , E, S_c and K_c are the thermal Grashof number, Solutal Grashof number, Magnetic parameter, Permeability parameter, Prandtl number, thermal radiation, Eckert number, heat absorption parameter, Schmidt number and chemical reaction parameter respectively.

The corresponding boundary conditions are

$$u = 0, \qquad \frac{\partial\theta}{\partial y} = -1, \qquad \frac{\partial C}{\partial y} = -1$$

at y = 0

$$u \to 0, \qquad \theta \to 0, \qquad C \to 0 \text{ as } y \to \infty \qquad (13)$$

3. SOLUTION OF THE PROBLEM

In order to solve the coupled nonlinear system of partial differential equations (10) to (12) with the boundary conditions (13) and (14), the following simple perturbation is used. The governing equations (10) to (12) are expanded in powers of Eckert number $E(<<1)$.

$$\left.\begin{aligned} &u = u_0 + Eu_1 + O(E^2),\ \theta = \theta_0 + E\theta_1 + O(E^2), \\ &C = C_0 + EC_1 + O(E^2) \end{aligned}\right\} \qquad (14)$$

Substituting equations (16) into equations (11) to (13) and equating the coefficients of the terms with the same powers of E, and neglecting the terms of higher order, the following equations are obtained.

Zero-th order terms:

$$u_0'' + u_0' = -G_r\theta_0 - G_m C_0 + M_1 u_0 \qquad (15)$$

$$\theta_0''+P_r\theta_0'-F\theta_0=0 \tag{16}$$

$$C_0''+S_C C_0'=S_C K_C C_0 \tag{17}$$

First order terms:

$$u_1''+u_1'=-G_r\theta_1-G_m C_1+M_1 u_1 \tag{18}$$

$$\theta_1''+P_r\theta_1'-F\theta_1=-\mathrm{P_r}\,u_0'^2 \tag{19}$$

$$C_1''+S_C C_1'=S_C k_C C_1 \tag{20}$$

The corresponding boundary conditions are

$$\left.u_0=0,\ u_1=0,\frac{\partial\theta_0}{\partial y}=-1,\ \frac{\partial\theta_1}{\partial y}=0,\frac{\partial C_0}{\partial y}=-1,\ \frac{\partial C_1}{\partial y}=0\right\}\quad \text{at } y=0$$

$$\left.u_0\to 0,\ u_1\to 0,\theta_0\to 0,\ \theta_1\to 0,C_0\to 0,\ C_1\to 0\right\}\quad \text{as } y\to\infty \tag{21}$$

Solving equations (15) to (20) under the boundary conditions (21)), the following solutions are obtained.

$$C_0=s_1 e^{-k_1 y} \tag{22}$$

$$\theta_0=s_2 e^{-k_2 y} \tag{23}$$

$$u_0=(-k_3-k_4)e^{-l_1 y}+k_3 e^{-k_2 y}+k_4 e^{-k_1 y} \tag{24}$$

$$\begin{aligned}u_1&=k_{18}e^{-2k_2 y}+k_{19}e^{-2k_1 y}+k_{20}e^{-2l_1 y}+k_{21}e^{-l_2 y}+k_{22}e^{-l_3 y}\\&+k_{23}e^{-l_4 y}+s_{12}e^{-l_6 y}+k_{25}e^{-k_1 y}-s_{13}e^{-l_7 y}\end{aligned} \tag{25}$$

$$\begin{aligned}\theta_1&=k_{11}e^{-2k_2 y}+k_{12}e^{-2k_1 y}+k_{13}e^{-2l_1 y}+\\&k_{14}e^{-l_2 y}+k_{15}e^{-l_3 y}+k_{16}e^{-l_4 y}+s_{11}e^{-l_6 y}\end{aligned} \tag{26}$$

$$C_1=0 \tag{27}$$

$$\theta=s_2 e^{-k_2 y}+E\begin{pmatrix}k_{11}e^{-2k_2 y}+k_{12}e^{-2k_1 y}+k_{13}e^{-2l_1 y}+\\ k_{14}e^{-l_2 y}+k_{15}e^{-l_3 y}+k_{16}e^{-l_4 y}+s_{11}e^{-l_6 y}\end{pmatrix} \tag{28}$$

$$C=s_1 e^{-k_1 y} \tag{29}$$

$$\begin{aligned}u&=(-k_3-k_4)e^{-l_1 y}+k_3 e^{-k_2 y}+k_4 e^{-k_1 y}+\\ &E\begin{pmatrix}k_{18}e^{-2k_2 y}+k_{19}e^{-2k_1 y}+k_{20}e^{-2l_1 y}+k_{21}e^{-l_2 y}+k_{22}e^{-l_3 y}\\ +k_{23}e^{-l_4 y}+s_{12}e^{-l_6 y}+k_{25}e^{-k_1 y}-s_{13}e^{-l_7 y}\end{pmatrix}\end{aligned} \tag{30}$$

1.5 NUSSELT NUMBER:

From temperature field, the rate of heat transfer in terms of Nusselt number is given in non-dimensional form as follows:

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=1 \tag{31}$$

1.6. SHERWOOD NUMBER:

From concentration field, the rate of mass transfer in terms of Sherwood number is given in non-dimensional form as follows:

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = 1 \qquad (32)$$

1.7. SKIN-FRICTION:

From velocity field, rate of change of velocity at the plate in terms of Skin-friction is given in non-dimensional form as follows:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\begin{aligned}\tau &= (k_3 + k_4)l_1 - k_3k_2 - k_4k_1 + \\ &E(-2k_2k_{18} - 2k_1k_{19} - 2l_1k_{20} - l_2k_{21} \\ &-l_3k_{22} - l_4k_{23} - l_6s_{12} - k_1k_{25}e^{-k_1y} + l_7s_{13}\end{aligned} \qquad (33)$$

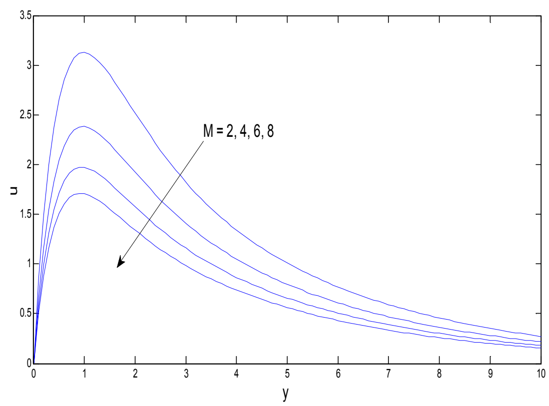

Figure.1. Effect of Magnetic parameter M on u

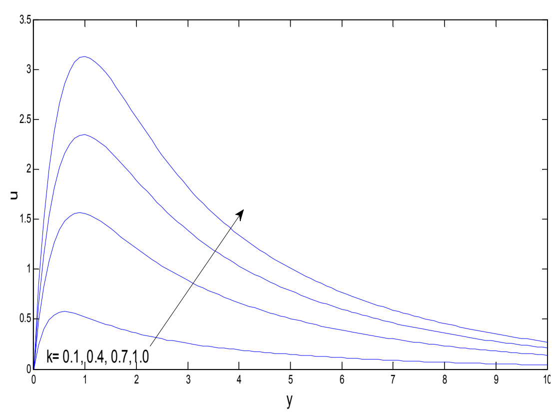

Figure.2. Effect of Permeability parameter k on u

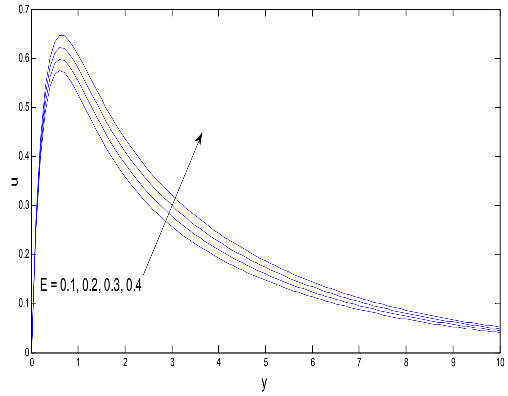

Figure.3. Effect of Eckert Number on u

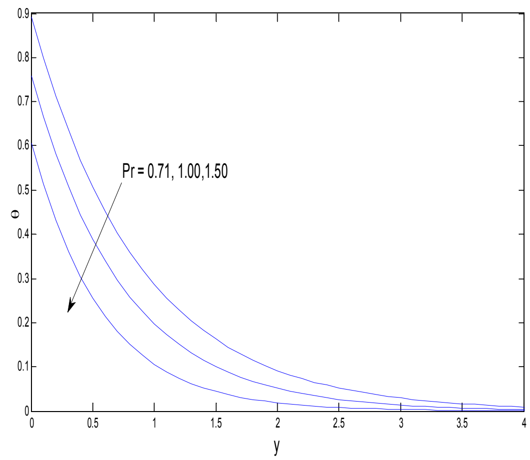

Figure.4. Effect of Prandtl Number P_r on temperature field

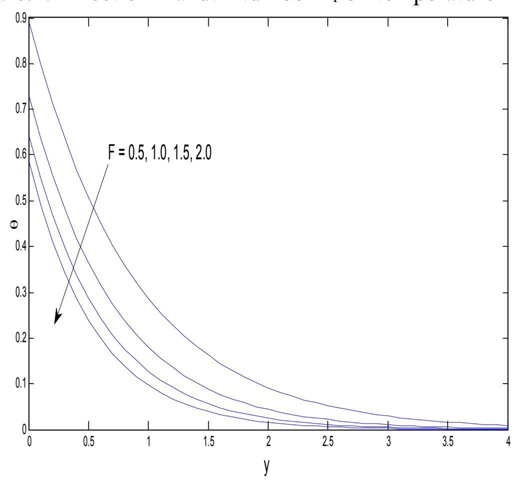

Figure.5. Effect of radiation parameter F on temperature field

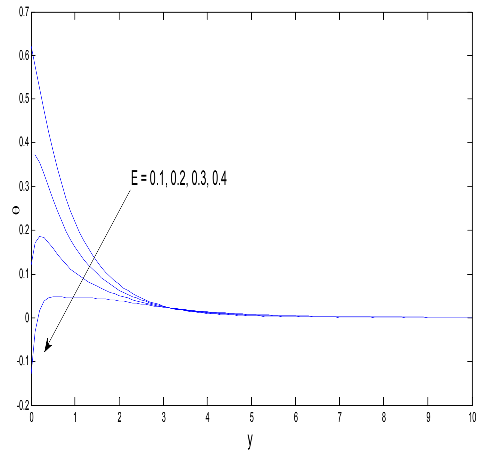

Figure.6. Effect of Eckert Number E on temperature field

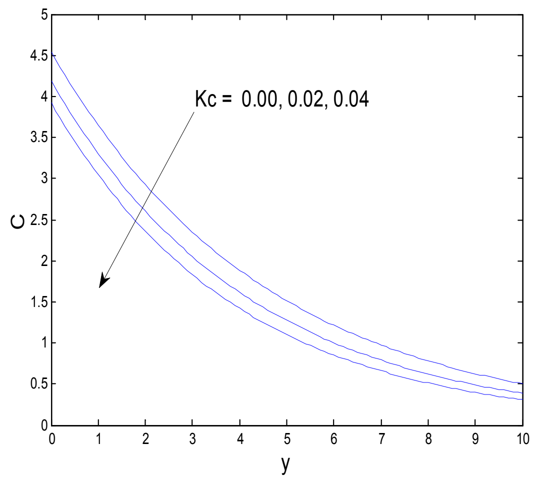

Figure. 7. Effect of chemical reaction parameter K_c on concentration field

RESULTS AND DISCUSSION

In order to get physical insight into the problem we have plotted velocity, temperature and concentration for various values of physical parameters from figures 1-8 Figure 15 reveals the velocity profiles for various values of magnetic parameter M. From this figure it is seen that with the increase in M the velocity decreases. The effect of permeability parameter k is pointed out in figure 2. From this figure it is seen that velocity increases with the increase of permeability of the porous medium. It is also noticed that from the figure 3, the effects of Eckert number E on velocity field, as E increases velocity increases.

The effects of various flow parameters on temperature field are exhibited through figures 4-6 respectively. It is observed that temperature enhances with increasing values of E while decreases with increasing values of P_r and F. Concentration profiles for different values of chemical reaction parameter K_c are shown in figures 7. It is observed that concentration increases with decreasing values of K_c S_c.

CONCLUSIONS

This manuscript investigates, the effect of radiation on MHD frees convection viscous dissipative past a moving vertical porous surface with chemical reaction was considered. The conclusions of the study are as follows:

1. The velocity decreases with increase values of magnetic parameter M or radiation parameter F or chemical reaction parameter K_c or Prandtl number P_r.
2. Temperature increases as the permeability parameter k increases while it decreases as radiation parameter F increases.
3. Concentration decreases with increasing chemical reaction parameter K_c .

SCOPE FOR FUTURE WORK

This study was conducted for a viscous electrically conducting Newtonian fluid. This work can be extended for the cases of well-known non-Newtonian fluids such as Casson fluid, micro-polar fluid, visco-elastic fluid etc.

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